Interplanetary Departure From Circular Earth Parking Orbits (EPOs)

Consider a spacecraft initially in a circular Earth parking orbit (EPO) with height "H" above Earth's equatorial radius R_E = 6378.136 km. Two impulsive maneuver strategies are to be compared in departing this EPO for interplanetary space, assuming unperturbed conic geocentric motion (Earth's reduced mass, μ = 398,600.44 km^3/s^2). The first strategy, called "direct departure" (DD), simply increases EPO speed to achieve the specified Earth departure asymptotic speed v_{HE} (v_{HE}^2 is equivalent to Earth departure energy "C3"). The second strategy, called "Earth gravity assist" (EGA), is more complex than DD. Using EGA, a retrograde impulse first establishes a departure elliptical orbit (DEO) with apsis heights H and H_{MIN}, where H_{MIN} is the lowest practical orbit height for safe navigation and acceptable aerodynamic energy losses from a single perigee passage. All EGA data presented herein reflect H_{MIN} = +200 km. A posigrade impulse is then applied at DEO perigee to achieve v_{HE} and complete an EGA departure.

In comparing impulsive DD costs with those of EGA, the following quantities and formulae (obtained form conic section geometry and the energy integral) are relevant.

1) $r_{EPO} \equiv$ geocentric EPO radius = $R_E + H$

2) $v_{EPO} \equiv$ geocentric speed in the EPO = $\sqrt{\frac{\mu}{r_{EPO}}}$

3) $v_{DD} \equiv$ required geocentric speed to achieve v_{HE} at r_{EPO} = $\sqrt{C3 + \frac{2\mu}{r_{EPO}}}$

4) $\Delta v_{DD} \equiv$ impulse required to achieve v_{DD} directly from EPO (this will be a positive value to indicate a posigrade impulse) = $v_{DD} - v_{EPO}$

5) $a_{DEO} \equiv$ semi-major axis of DEO = 0.5 ($r_{EPO} + r_{MIN}$)

6) $v_1 \equiv$ geocentric DEO speed at r_{EPO} = $\sqrt{\mu\left(\frac{2}{r_{EPO}} - \frac{1}{a_{DEO}}\right)}$

7) $\Delta v_1 \equiv$ impulse required to transfer from EPO to DEO apogee (this will typically be a negative value to indicate a retrograde impulse) = $v_1 - v_{EPO}$

8) $v_2 \equiv$ geocentric DEO speed at r_{MIN} = $\sqrt{\mu\left(\frac{2}{r_{MIN}} - \frac{1}{a_{DEO}}\right)}$

9) $v_{EGA} \equiv$ required geocentric speed to achieve v_{HE} at r_{MIIN} = $\sqrt{C3 + \frac{2\mu}{r_{MIN}}}$

10) $\Delta v_2 \equiv$ impulse required to achieve v_{EGA} from DEO perigee (this will typically be a positive value to indicate a posigrade impulse) = $v_{EGA} - v_2$

11) $\Delta v_{EGA} \equiv$ total impulse required to achieve v_{EGA} from EPO after transfer to DEO perigee (this will be a positive value) = $|\Delta v_1| + |\Delta v_2|$

12) $\Delta\Delta v \equiv$ EGA impulse margin with respect to DD impulse = $\Delta v_{DD} - \Delta v_{EGA}$

Therefore, EGA is impulsively more efficient than DD only when $\Delta\Delta v$ is positive. Furthermore, because EGA requires loiter through at least half a DEO before Earth departure is achieved, DD is also clearly the superior choice from a timeline perspective when $\Delta\Delta v$ is negative or marginally positive.

For the parabolic escape case with $v_{HE} = 0$, Figure 1 plots $\Delta\Delta v$ as a function of H. There is no EPO for which Δv_{EGA} is less than Δv_{DD} in this case.

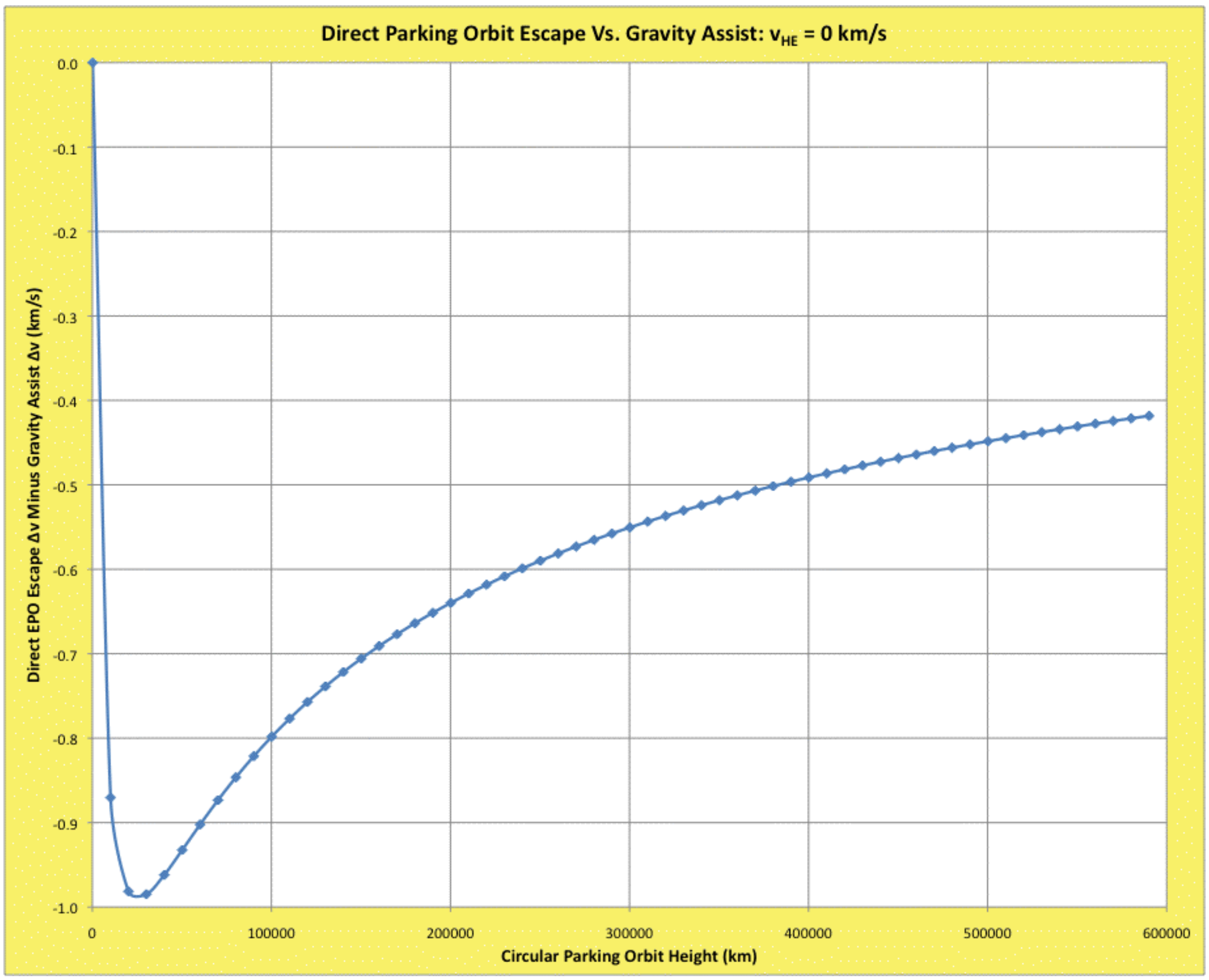

Figure 1: $\Delta\Delta v$ Versus H for Parabolic Earth Departure v_{HE} = 0

When v_{HE} is increased to 2 km/s, the situation changes dramatically. As illustrated by Figure 2, Δv_{EGA} is less than Δv_{DD} when H exceeds +200,000 km, about halfway to the Moon's orbit.

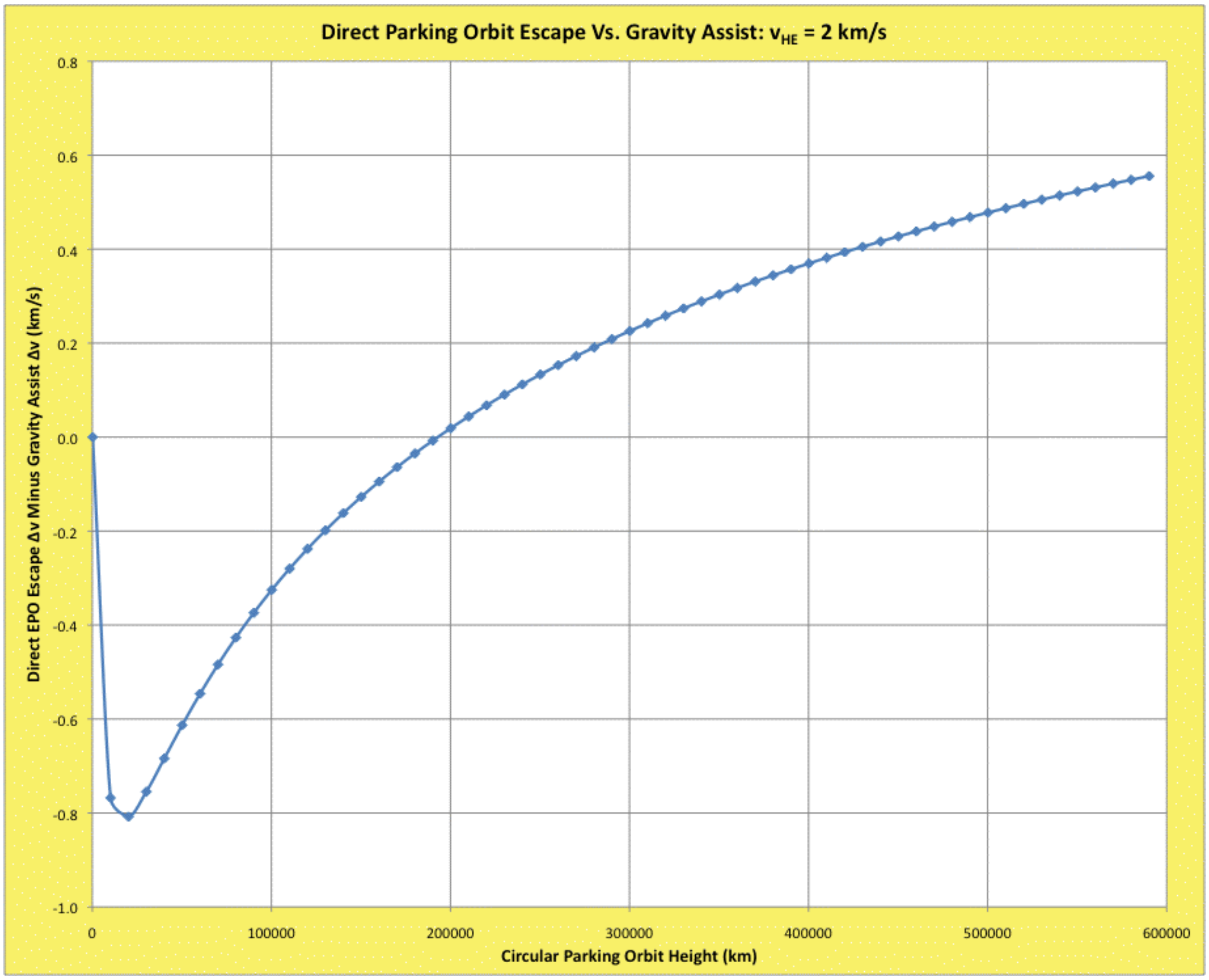

Figure 2: ΔΔv Versus H for Earth Departure v_{HE} = 2 km/s

Although conventional coasted trajectories to Venus and Mars typically call for v_{HE} well above 2 km/s, the most accessible near-Earth objects (NEOs) may require v_{HE} considerably less than 2 km/s. In these nearly parabolic Earth departures, a DD strategy may be preferable to EGA, even if EPO is in cis-lunar space with H near +400,000 km.